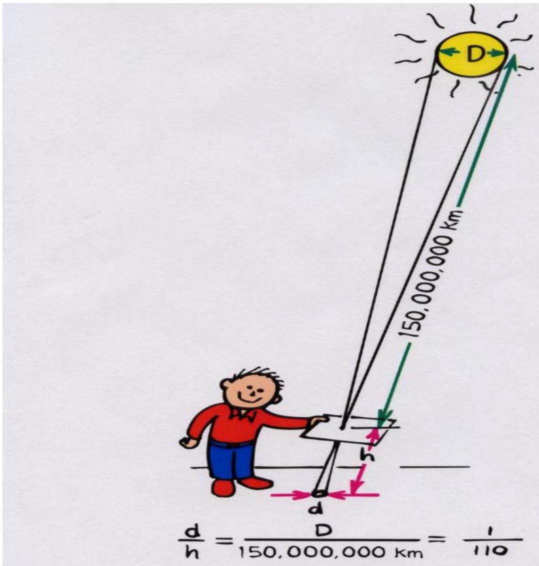
Measuring the Size of the Sun

by Robin Riordan

The total lunar eclipse of October 28 brings us to the second part of the adventure of determining the diameter of the Sun. Last month we used a pinhole projector to determine the ratio of the Sun's diameter to its distance. Follow is a diagram of the setup.



Hewitt, *Conceptual Physics*, Ninth Edition.

With my ideal pinhole camera measured the diameter of the Sun's image as 3mm. The distance between the pinhole and the screen was 330mm.Remember the ratio:

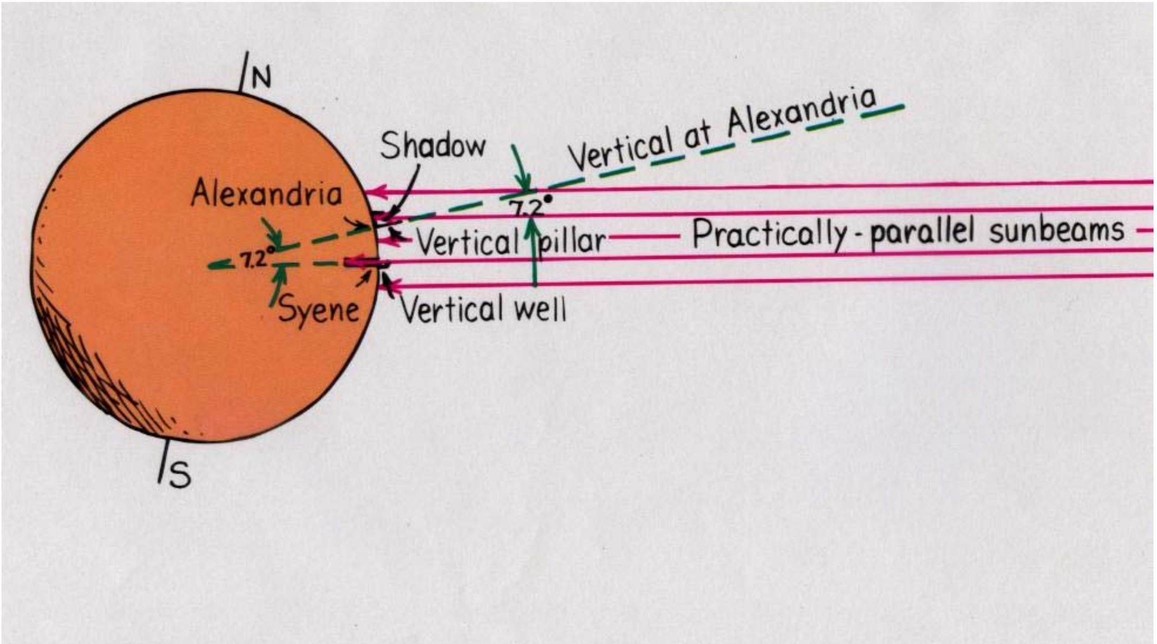
$$\frac{\text{image distance}}{\text{projection diameter}} = \frac{\text{Sun's distance}}{\text{Sun's diameter}}$$

so

$$\frac{330\text{mm}}{3\text{mm}} = 110$$

So our equation becomes:
Sun's distance = 110 * Sun's diameter
All we need is to either out the Sun's diameter or its distance to get the other value. This is where the eclipse comes in. But first let's back up to another brilliant piece of science by Erastosthenes. We need this experiment to make use of the eclipse.

Around 235 BC Erastosthenes measured the size of the Earth. He had learned from the archives in the Great Library of Alexanderia that the Sun appeared directly overhead on the summer solstice in an Egyptian town called Syene. In fact, one could see the image of the Sun at noon reflected directly back upward from a deep well there. You and I know that this means that Syene must be located at 23.5 degrees north latitude. Erastosthenes measured the angle of the Sun's shadow cast on the same day in his town of Alexandria as 7.2 degrees. He also knew that the distance between Syene and Alexandria was 800 km north (along the same arc of longitude). Great! He now had a triangle. Let's look.



Hewitt, *Conceptual Physics*, Ninth Edition.

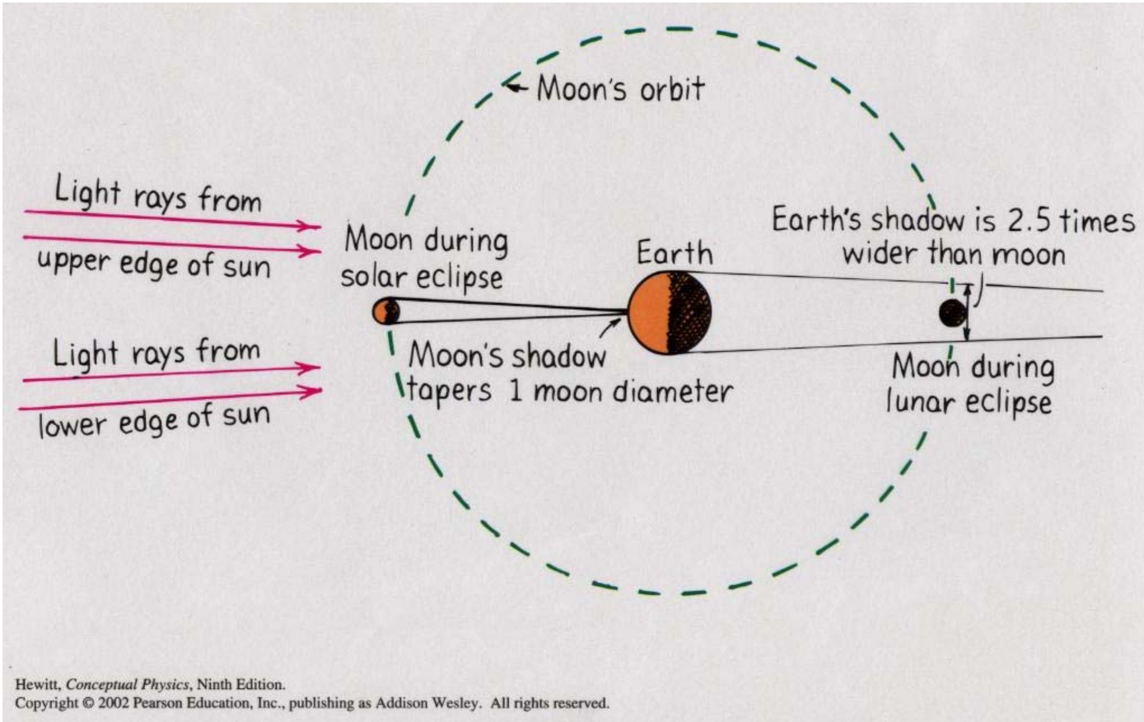
This distance of 800 km corresponds to 7.2 degrees. So, What distance corresponds to 360 degrees(a full circle ... circumference)?

$$\frac{\text{Circumference}}{800\text{km}} = \frac{360 \text{ degrees}}{7.2 \text{ degrees}}$$

So, the circumference = $\frac{800\text{km} * 360}{7.2}$ = 40,000km

This means the Earth's diameter is 40,000 km/3.14 = 12,740km

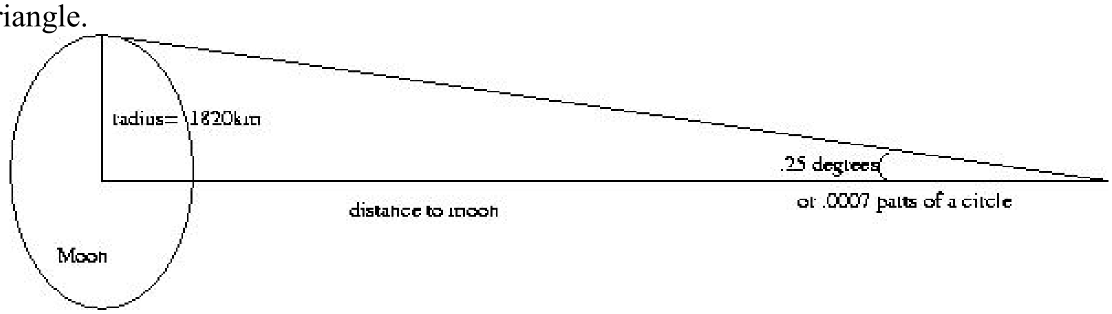
Now we can use the known size of the Earth and cast a shadow onto the Moon. By timing the length of time it takes the Earth's shadow to cross the Moon, and taking into account that the Sun's rays are slanted, because the Sun is not a point source (this is a detail), we deduce that the Earth's diameter is 3.5 times that of the Moon. Following is a diagram.



I know the diagram says Earth's shadow is 2.5 times wider that the Moon but you get a better approximation if you take into account the slant of the Sun's rays. So now, by division, we calculate the Moon's diameter.

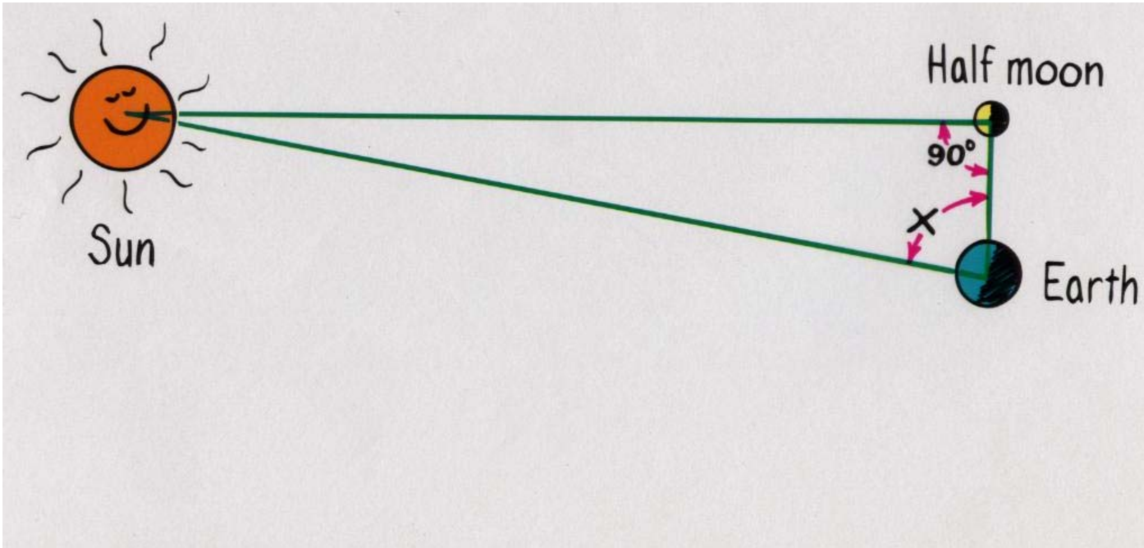
$$\text{Moon Diameter} = \frac{\text{Earth diameter}}{3.5} = \frac{12{,}740\text{km}}{3.5} = 3640\text{km (about)}$$

Let's see what we have so far. 1) ratio of the Sun's distance to its diameter and 2) the diameter of the Moon. Yes, we went through all this to get these two bits of information. We now need to know the distance to the moon . This one is not too bad. Another triangle.



From this triangle we calculate the Moon's distance as approximately 384,000km. Now bear in mind that these values are neither precise. That are merely to illustrate.

The last triangle we need is that between the Moon, Earth, and Sun. We can determine the distance to the Moon using another triangle and a quarter (half) Moon. Here is another diagram.



This angle X is a tough measurement because the Sun and the Moon are not easy pinpoint sources. You have to decide where the centres are. As it turns out angle X is about 89.4 degrees. Great! We have the distance between the Earth and Moon, and the angle between the Moon-Earth-Sun. Using some trigonometry we calculate the distance to the Sun as 150 million km.

Finally, back to our little pinhole. The ratio of the sun's distance to its diameter was 110. So,

$$\frac{\text{Sun's Distance}}{\text{Sun's Diameter}} = 110$$

$$\text{Sun's Diameter} = \frac{\text{Sun's Distance}}{110} = \frac{150 \text{ million km}}{110} = 1.36 \text{ million km}$$

Like any science, Astronomy requires solving many smaller problems and bringing the results together to answer the original question. Those of us who are attracted to astronomy are naturally inclined problem solvers. The pieces of the puzzle are the fun. This is where art and science come together. The common bond is creativity. The artist solves the problem with stone, the scientist with logic. The artist treats the question with paint, the scientist with mathematics. The artist arrives at solutions with sound, the scientist (Gil and others) with duct tape. What a wonderful enterprise.